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COS 488
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PS4 - Q4

From the previous question, we had that the EGF for permutations whose cycles are all of odd length is

$$O(z) = \sqrt{\frac{1+z}{1-z}} = \frac{1}{1-z} * \sqrt{1-z^2}$$

Note that $[z^N]O(z) = \frac{O_N}{N!}$ is precisely the probability we are looking for.

We get coefficients of the last term by the binomial expansion:

$$[z^N](\sqrt{1-z^2}) = \binom{\frac{1}{2}}{\frac{N}{2}}(-1)^{N/2}$$

For even values of N, and $[z^N](\sqrt{1-z^2}) = 0$ for N odd.

Convoluting this with $\frac{1}{1-z}$ gives, for even values of N:

$$[z^N]\left(\frac{1}{1-z} * \sqrt{1-z^2}\right) = \sum_{k=0,2,\ldots N} \binom{\frac{1}{2}}{\frac{k}{2}}(-1)^{k/2} = \sum_{0 \le k \le N/2} \binom{\frac{1}{2}}{k}(-1)^k$$

$$= \frac{1}{\sqrt{\pi * \frac{N}{2}}}$$

I do not see how this last step follows.

as desired and hence we have shown the result.

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