COS 488 Week 4: Q1

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5/5

5.1) How many bitstrings of length N have no 000?

$$B_{000} = E + Z_0 + Z_0 \times Z_0 + (Z_1 + Z_0 \times Z_1 + Z_0 \times Z_0 \times Z_1) \times B_{000}$$

Or, in English: "a binary string with no 000 is either empty, or 0, or 00, or it is a 1, 01, or 001, followed by a binary string with no 000." Thus the OGF equation is:

$$B_{000}(z) = 1 + z + z^2 + (z + z^2 + z^3)B_{000}(z)$$

which has solution:

$$B_{000} = \frac{1 + z + z^2}{1 - z - z^2 - z^3}$$

Now to find an asymptotic approximation, we use the rational functions transfer theorem on slide 44 of AA05-AC.pdf. We see that $f(z) = 1 + z + z^2$ and $g(z) = 1 - z - z^2 - z^3$, so $g'(z) = -1 - 2z - 3z^2$. We solve for the roots of g, and get that they are approximately $z = .5437$ and $z = -.7718 \pm 1.1151i$, the latter of which has modulus 1.3562, thus the real solution is closest pole to the origin. Thus if we plug into the rational functions transfer theorem, we get:

$$((-1/.5437)(1+.5437+.5437^2)/(-1-(2\cdot .5437)-(3\cdot .5437^2))(1/.5437)^N = 1.137(1.84)^N$$

(Worked with Maryam B.)