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COS 488 Week 4: Q3

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5.7) Derive an EGF for the number of permutations whose cycles are all of odd length. First we look for a generating function for the number of odd cycles over different n. To do this, we first note an equality between the number of cycles on n elements, and the number of cycles for n odd and n even:

$$\sum_{k\geq 0}\frac{z^k}{k} = \sum_{k\geq 0}\frac{z^{2k}}{2k} + \sum_{k\geq 0}\frac{z^{2k+1}}{2k+1}$$

$$= \frac{1}{2}\sum_{k\geq 0}\frac{(z^2)^k}{k} + \sum_{k\geq 0}\frac{z^{2k+1}}{2k+1}$$

Thus we can solve for just the odd case, getting:

$$CYC_{odd}(k) = \sum_{k\geq 0}\frac{z^k}{k} - \frac{1}{2}\sum_{k\geq 0}\frac{(z^2)^k}{k} = \ln\frac{1}{1-z} - \frac{1}{2}\ln\frac{1}{1-z^2} = \ln\frac{\sqrt{1-z^2}}{1-z}$$

Now, since we want $SET(CYC_{odd}(k))$, we must take the exponential of our expression, giving:

$$e^{\ln\frac{\sqrt{1-z^2}}{1-z}} = \frac{\sqrt{1-z^2}}{1-z} = \sqrt{\frac{1+z}{1-z}}$$

(Worked with Maryam B.)