COS 488 Week 4: Q4

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5/5

5.23) Show that the probability that all of the cycles are of odd length in a random permutation of length N is $1/\sqrt{\pi N/2}$ (see Exercise 5.7).
We note that our answer to 5.7 was:

$$\sqrt{\frac{1+z}{1-z}}$$

Thus we can apply the transfer theorem (note that the numerator has an infinite radius of convergence) with $\rho = 1$, $\alpha = 1/2$, and $f(z) = \sqrt{1+z}$. This gives:

$$\frac{\sqrt{1+1}}{\sqrt{\pi}} 1^{-N} N^{-1/2} = 1/\sqrt{\pi N/2}$$

We have to multiply by $N!$ to adjust for the method of constructing the generating function, but then we also need to divide by $N!$ to adjust for taking the probability over all permutations (of which there are $N!$), leaving our answer as the final solution, as desired.
(Worked with Maryam B.)