Homework 4: Exercise 5.7

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<p style="color:red; text-align:center">5/5</p>

Let $\mathcal{P}$ be the class of permutations, in which all cycles have odd length. $\mathcal{P}$ can be specified symbolically as

$$\mathcal{P} = \text{SET}(\text{CYC}_{\text{odd}}(\mathcal{Z})).$$

Since the exponential generating function for cycles of size k is z^k/k, we have following generating function for $\mathcal{P}$:

$$
\begin{aligned}
P(z) &= \exp\left(\sum_{\text{odd } k} \frac{z^k}{k}\right) \\
&= \exp\left(\sum_{k \geqslant 0} \frac{z^{2k+1}}{2k+1}\right) \\
&= \exp\left(\sum_{k \geqslant 1} \frac{z^k}{k} - \sum_{k \geqslant 1} \frac{z^{2k}}{2k}\right) \\
&= \exp\left(\ln\left(\frac{1}{1-z}\right) - \frac{1}{2}\ln\left(\frac{1}{1-z^2}\right)\right) \\
&= \frac{\frac{1}{1-z}}{\sqrt{\frac{1}{1-z^2}}} = \frac{\sqrt{1-z^2}}{1-z} = \sqrt{\frac{1+z}{1-z}}
\end{aligned}
$$