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# COS 488 - Homework 4 - Question 3

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Let $\mathcal{O}$ be the class of all permutations whose cycles are all of odd length with $|p|$ being the number of atoms in $p$ for each $p \in \mathcal{O}$. Let $Z$ denote a labelled atom, which has size 1.
Let $CYC_{\mathcal{O}}(Z)$ be the class of all labelled cycles of $Z$ of odd length, so that

$$CYC_{\mathcal{O}}(Z) = \frac{CYC(Z) - CYC(-Z)}{2} = \frac{\ln \frac{1}{1-Z} - \ln \frac{1}{1+z}}{2} = \ln \sqrt{\frac{1+Z}{1-Z}}.$$

^nice derivation

Then, an element of  is a sequence of elements of $CYC_{\mathcal{O}}(Z)$, so we have the construction

$$\mathcal{O} = SEQ(CYC_{\mathcal{O}}(Z)),$$

which gives the EGF equation

$$\mathcal{O}(z) = e^{\ln \sqrt{\frac{1+Z}{1-Z}}} = \sqrt{\frac{1+Z}{1-Z}}.$$