COS 488 - Homework 4 - Question 4

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From problem 3, the number of permutations $\mathcal{O}_N$ of length N having all cycles of odd length is the coefficient of $\frac{z^N}{N!}$ in the expansion of $\sqrt{\frac{1+Z}{1-Z}}$. Therefore, since there are $N!$ permutations of length N, the probability that all of the cycles of a random permutation are odd is $\frac{\mathcal{O}}{N!}$, which is the coefficient of z^N in the expansion of $\sqrt{\frac{1+Z}{1-Z}}$. Thus, by the radius-of-convergence theorem, we have that this probability is

$$[z^N]\sqrt{\frac{1+Z}{1-Z}} \sim \frac{\sqrt{1+1}}{\Gamma\left(\frac{1}{2}\right)} N^{\frac{1}{2}-1} = \frac{1}{\sqrt{\pi N/2}},$$

as desired.