COS 488 Problem Set #4 Question #1

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If $\mathcal{B}$ is the set of bitstrings not containing 000, then we have the following combinatorial construction for $\mathcal{B}$:

$$\mathcal{B} = \epsilon + \mathcal{Z}_0 + \mathcal{Z}_1 \times \mathcal{B} + \mathcal{Z}_0 \times \mathcal{Z}_0 + \mathcal{Z}_0 \times \mathcal{Z}_1 \times \mathcal{B} + \mathcal{Z}_0 \times \mathcal{Z}_0 \times \mathcal{Z}_1 \times \mathcal{B}$$

As a result, we have the following formula for the generating function:

$$B(z) = 1 + z + z^2 + (z + z^2 + z^3)B(z)$$
$$B(z) = \frac{1 + z + z^2}{1 - z - z^2 - z^3}$$

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How many bitstrings are there? Exact or asymptotic answer?