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## COS 488 Problem Set #4 Question #2

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As noted in the book, we have the following construction for binary trees:

$$\mathcal{T} = \mathcal{Z}_{\square} + \mathcal{Z}_{\bullet} \times \mathcal{T} \times \mathcal{T}$$

If we are counting all nodes, then both  $\mathcal{Z}_{\square}$  and  $\mathcal{Z}_{\bullet}$  have EGF's of  $z$ . Hence, we have the equation

$$\begin{aligned} T(z) &= z + zT(z)^2 \\ zT(z)^2 - T(z) + z &= 0 \\ T(z) &= \frac{1 \pm \sqrt{1 - 4z^2}}{2z} \end{aligned}$$

We know  $T(0) = 0$ , so that fixes the minus sign since otherwise the expression would be undefined. As a result  $T(z) = \frac{1 - \sqrt{1 - 4z^2}}{2z}$ .