

COS 488 Problem Set #4 Question #3

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Let \mathcal{D} denote the combinatorial class of all permutations consisting only of odd-length cycles. Then in particular we have the following:

$$\begin{aligned}\mathcal{D} &= SEQ(CYC_1 + CYC_3 + \dots + CYC_{2n+1} + \dots) \\ D(z) &= \exp(z + z^3/3 + \dots + z^{2n+1}/(2n+1) + \dots)\end{aligned}$$

Note that $-\frac{1}{2} \log(1 - z^2) = z^2/2 + z^4/4 + \dots + z^{2n}/(2n)$. As a result, the expression in the exponential reduces to $-\log(1 - z) + \frac{1}{2} \log(1 - z^2)$. Hence,

$$\begin{aligned}D(z) &= \exp\left(\frac{1}{2} \log(1 - z^2) - \log(1 - z)\right) \\ &= \frac{\sqrt{1 - z^2}}{1 - z} \\ &= \sqrt{\frac{1 + z}{1 - z}}\end{aligned}$$