COS 488 Problem Set #4 Question #4

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March 2, 2017

5/5

It's well-known that the binomial expansion $(1+x)^\alpha$ converges absolutely at the endpoints of its radius of convergence (which is 1) when $\alpha > 0$. Since the proof of Theorem 5.5 only requires that the sum of the f_i converge to f, this suffices for theorem 5.5 to $f(z) = \sqrt{1+z}$. If we choose $\alpha = 1/2$, then we have

$$[z^N]\sqrt{\frac{1+z}{1-z}} \sim \frac{\sqrt{2}}{\sqrt{\pi N}}$$

We know from the previous problem that this is the EGF for permutations composed of odd cycles. Since permutations have EGF of $1/(1-z)$ which simply has coefficients of 1 for each z^N, it follows that the proportion of permutations that decompose into odd cycles tends to $1/\sqrt{\pi N/2}$ for N arbitrarily large.