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COS 488 Week 5: Q3

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7.29) An *arrangement* of N elements is a sequence formed from a subset of elements. Prove that the EGF for arrangements is $e^z/(1-z)$. Express the coefficients as a simple sum and interpret that sum combinatorially.
We note that an arrangement of N elements is a sequence formed from a subset of the elements. Thus, it consists of an ordered pair of: a set of atoms that we don't permute, and a set of atoms that we do. Thus we have:

$$SET(z) \star PERM(z) = SET(z) \star SET(CYC(z))$$

$$\implies e^z * e^{\ln \frac{1}{1-z}} = \frac{e^z}{1-z}$$

as desired. We note that this is the EGF convolution (binomial convolution) of $1/(1-z)$ and e^z, the first of which is just the sequence of factorials, and the second of which is just a sequence of 1s. Thus if we look at the binomial convolution formula:

$$\sum_{n\geq 0} \sum_{0\leq k\leq n} (\binom{n}{k} a_k b_{n-k}) z^n/n!$$

we see that we can let every b_{n-k} be 1, and that the other term will always be k!. Thus we see that the coefficients are just the simple sum

$$\sum_{0\leq k\leq n} \binom{n}{k} k!$$

or, relabeling:

$$\sum_0 n!/k!$$

. Combinatorially, we can think of this in several ways. It is all of the different ways you can choose then permute a subset of n items (of any size). Or you can think of it as all of the ways to permute n items but where any number between 1 and n of them can be identical.
Worked with Eric N.

the purpose of the interpretation was to explain how one could derive the result w/o using GFs, your second interpretation is not really clear in this light