COS 488 - Homework 5 - Question 3

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An arrangement of N elements consists of a subset of $\{1, 2, \ldots, N\}$ that is un-ordered and a subset that is ordered. Therefore, if A is the class of ordered arrangements, then we have the construction

$$A = SET(z) \star SEQ(z),$$

which gives the EGF equation

$$A(z) = \frac{e^z}{1 - z}.$$

By explicitly computing terms of this convolution, we have that

$$[z^n]A(z) = \sum_{k=0}^{n} \binom{n}{k} k!,$$

which makes sense because there are $\binom{n}{k}$ ways to choose a k-element subset of n items and $k!$ ways to arrange those elements, so by multiplying and summing over k, we have the total number of arrangements.

good, concise explanation