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COS 488 Problem Set #5 Question #3

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If $\mathcal{A}(\mathcal{Z})$ is the set of arrangements, then we have that $\mathcal{A}(\mathcal{Z}) = \mathcal{P}(\mathcal{Z}) \times SET(CYC_1(\mathcal{Z}))$, since every arrangement of n letters consists of fixing k letters and then permuting the remaining $n-k$ letters arbitrarily. This gives

$$\begin{aligned}
A(z) &= \frac{e^z}{1-z} \\
&= \left(\sum_{n=0}^{\infty} \frac{z^n}{n!}\right)\left(\sum_{n=0}^{\infty} z^n\right) \\
&= \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} \frac{1}{k!} z^n\right) \\
&= \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} \frac{n!}{k!} \frac{z^n}{n!}\right) \\
&= \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n} \binom{n}{k}(n-k)! \frac{z^n}{n!}\right)
\end{aligned}$$

Note that to find the number of arrangements we can choose k elements to fix and then permute the remaining $n-k$ elements, giving $\sum_{k=0}^{n} \binom{n}{k}(n-k)!$ arrangements of n letters.