5/5

David Luo
Exercise 8.3

"How long a string of random bits should be taken to be 50% sure that there are at least 32 consecutive 0s?"

We are going to need a formula for N when $P(\textit{at least } 0^{32}) = 1 - P(\textit{no } 0^{32}) = 1/2$, or N when $P(\textit{no } 0^{32}) = 1/2$

Invoke the theorem from lecture that the probability an N-bit random bitstring has no 0^k is $[z^N]S_k(z/2) \sim c_k(\beta_k/2)^N$

$S_{32}(z) = \frac{1-z^{32}}{1-2z+z^{33}}$, $\beta_{32} = 1/root(1-2z+z^{33}) \sim 1.9999999997671693554787686145172414509473202231021622632 74...$

$c_{32} \sim \frac{-\beta_{32}(1-\frac{1}{\beta_{32}{}^{32}})}{-2+33(\frac{1}{\beta_{32}})^{32}} = 1.0000000034924596812625776606680637236657320998141902533720348...$

$P(\textit{no } 0^{32}) = [z^N]S_{32}(z/2) \sim c_{32}(\beta_{32}/2)^N = 1/2$

$N = log_{\beta_{32}/2}(\frac{1}{2c_{32}}) \sim 5.954 \times 10^9$