David Luo
Exercise 8.14

5/5

"Suppose that a monkey types randomly at a 32-key keyboard. What is the expected number of characters typed before the monkey hits upon the phrase THE QUICK BROWN FOX JUMPED OVER THE LAZY DOG?"

This is a 44 character sequence. Define this sequence as the pattern p.

Now define S_p as 32-key strings without p and T_p as strings that end in p but have no other occurrence of p.
We have two resulting constructions: One from the notion that these are two disjoint sets and that adding a character to a string in S_p gives a string in one of the two sets, and another from the notion that adding certain character sequence "tails" to strings in T_p results in strings that are made up of another string in S_p and p itself. These are:

$$S_p + T_p = E + S_p \times \{Z_0 + ... + Z_{31}\} \qquad S_p \times \{p\} = T_p \times \sum_{c_i \neq 0} \{t_i\}$$

which result in the OGFs:

$$S_p(z) + T_p(z) = 1 + 32zS_p(z) \qquad S_p(z)z^{|p|} = T_p(z)c_p(z)$$

Solve for $S_p(z)$ by first removing T:

$$T_P(z) = 1 + (32z - 1)S_p(z) = \frac{S_p(z)z^p}{c_p(z)}$$

$$S_p(z)(z^p + (1 - 32z)c_p(z)) = c_p(z)$$

$$S_p(z) = \frac{c_p(z)}{z^p + (1 - 32z)c_p(z)}$$

$$= \frac{1}{z^{44} + 1 - 32z}$$

Note that

$$S_p(z) = \sum_{N \geq 0} \{\# \; of \; 32 \; key \; strings \; without \; p\} z^N$$

and that

$$S_p(1/32) = \sum_{N \geq 0} \{\# \; of \; 32 \; key \; strings \; without \; p\}/32^N = \sum_{N \geq 0} P(position \; of \; first \; p \; is \; after \; N) = E(position \; of \; ending \; of \; first \; p)$$

$$\sim 1.685 * 10^{66}$$