Analytic Combinatorics Homework 6 Problem 1

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3/14/2017

From Slide 8, we have that the generating function for the number of n-bit strings with no runs of 32 consecutive 0s is

$$B_{32}(z) = \frac{1 - z^{32}}{1 - 2z + z^{33}}.$$

The root of smallest magnitude of $z^{33} - 2z + 1$ is $\frac{1}{\beta} \approx 0.5000000000582$; this is a single root. Thus, from the Asymptotics lecture, we have that

$$[z^n]B_{32}(z) \sim \beta^n \frac{-\beta f\left(\frac{1}{\beta}\right)}{g'\left(\frac{1}{\beta}\right)},$$

where $f(z) = 1 - z^{32}$ and $g(z) = 1 - 2z + z^{33}$. Evaluating this expression gives

$$[z^n]B_{32}(z) \sim 1.00000000349 \cdot 1.999999999767^n.$$

The total number of n-bit strings is 2^n. Thus, we wish to know how large n must be such that

$$1.00000000349 \cdot \left(\frac{1.999999999767}{2}\right)^n < \frac{1}{2}.$$

Then

$$n \approx \frac{\ln \frac{1}{2 \cdot 1.00000000349}}{\ln \frac{1.999999999767}{2}} \approx \frac{\ln 0.499999998258}{\ln 0.999999999883} \approx \frac{-0.693}{-1.164 \times 10^{-10}} \approx \boxed{5.954 \times 10^9}.$$