COS 488 Week 6: Q1

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How long a string of random bits should be taken to be 50% sure that there are at least 32 consecutive 0s?
We want an N such that the number of strings with at least 32 consecutive zeros is half of the total number of strings (since each string occurs with equal probability). The total number of strings is 2^N.
From the slides, the number of N-bit binary strings with no runs of P=32 consecutive zeros is $c_k\beta_k^N$ where β is the dominant root of $1-2z+z^{33}$. Thus we apply the rational functions transfer theorem. $-\frac{\beta f(1/\beta)}{g'(1/\beta)}\beta^n$ where we use Mathematica to find that the root of smallest modulus is approximately z = .5000000000582. Thus we want

$$-\frac{\beta(1-(1/\beta)^{32})}{-2+33(1/\beta)^{32}} * \beta^n/2^n = .5$$

solving this, we get $n = 5.95 * 10^9$.