COS 488 Week 6: Q2

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2.5/5

Suppose that a monkey types randomly at a 32-key keyboard. What is the expected number of characters typed before the monkey hits upon the phrase THE QUICK BROWN FOX JUMPED OVER THE LAZY DOG?

First of all, I just want you to know that it upsets me that they took a perfectly good pangram and replaced "jumps" with "jumped." And now to the question...

True

Where does the Z<44 come from? Please explain this explicitly

We have the construction (note that $K_{32} = Z_{<44}(E + Z_1 K_{32} + Z_2 K_{32} + ... + Z_{31} K_{32})$ where Z_{32} is the character that would continue our sequence in a way that we want to avoid. Thus we get the OGF equation

$$K_{32}(z) = (1 + z + ... + z^{44})(1 + 31zK_{32}(z))$$

This includes strings that contain the pattern. For example if the prefix is THE and the next letter is T then an element of K_32 is HE QUICK BROWN FOX etc

with solution

Should be 1-z^45 from your previous step

$$K_{32}(z) = \frac{1 - z^{44}}{1 - 32z + 31z^{46}}$$

-0.5

Now, as is done in slide 10 in the "strings" lecture, we can go through the same manipulations to get that plugging in 1/32 will give us a sum over N of the number of bitstrings of length N with occurrences of the above phrase, divided by 32^N (which is the total number of strings of size N with a 32-character alphabet), and thus it gives the expected position of end of the first occurrence of the above phrase. Plugging in 1/32 we get $5.56 * 10^6 7$ expected typed characters.

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