COS 488 Week 6: Q4

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March 17, 2017

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Describe the graph structure of *partial* mappings, where the image of a point may be undefined. Set up the corresponding EGF equations and check that the number of partial mappings of size N is $(N+1)^N$.

We first note that we can think of the undefined map as an extra singleton cycle that the trees can possible map to. It is a singleton because the undefined map must always be sent to itself, under the tree-model. (It isn't actually one of our nodes/elements, but for this analogy to work, we must present it in this manner so as to create a bijection between the analogous scenarios.) Thus since we know that mappings are SET(CYC(C(z))), here we instead of SET(CYC(C(z)) + CYC_1(C(z))) since CYC(C(z)) normally represents cycles of size 1 or 2 or 3... so here we have cycles of size 1 or 1 or 2 or 3... I.e. we have an extra size-1 option, as required. Thus our EGFs are:

$$C(z) = ze^{C(z)}$$

and

$$M(z) = SET(CYC(C(z)+CYC_1(C(z))) = SET(CYC(C(z)+C(z)) = e^{\ln\frac{1}{1-Cz}+C(z)=e^{C(z)}/(1-C(z))}$$

Now we apply Lagrange-Bürmann inversion as is AA09-words, and see that if we let $f(u) = u/e^u$ and $H(u) = e^u/(1-u)$ and $g(z) = C(z)$ then our conditions are met and we have

$$[z^n]M(z) = \frac{1}{n}[u^{n-1}]\frac{e^u(u-2)}{(1-u)^2}e^{un} = \frac{u-2}{(1-u)^2}e^{u(n+1)}$$

from which we get (after shifting, since we added the "undefined" node earlier):

$$\sum_{0\le k\le N}(N+1-k)N^{k-1}/k! = \sum_{1\le k\le N}(N+1)^k/k! - \sum_{1\le k\le N}N^k/(k-1)! = (N+1)^N/N!$$

giving $(N+1)^N$ as the number of partial mappings. Note that this is also seen to be correct as we have N elements, each of which could go to N+1 spaces, thus $(N+1)^N$ should be the answer.