

Homework 6: Exercise 8.3

Maryam Bahrani (mbahrani)

5/5

David Luo

Let $B(z)$ be the generating function for binary strings with no runs of 32 zeros. As derived in lecture,

$$B(z) = \frac{1 - z^{32}}{1 - 2z + z^{33}}$$

Using the rational function transfer theorem and a symbolic math package (sage), we find the dominant root of the denominator is 0.500000000058358, so $\beta = 1/0.500000000058358 = 1.99999999976657$. The asymptotics are then

$$[z^N]B(z) = C\beta^N = C \cdot 1.99999999976657^N$$

$$C = -\beta \frac{1 - (1/\beta)^{32}}{-2 + 33(1/\beta)^{32}} = 1.00000000349216$$

Furthermore, we know that there are 2^N binary strings of length N . To be 50% sure a random binary string of length N has a run of 50 consecutive zeros, we need to pick N so that half the strings have runs of 32 consecutive zeros, *i.e.*,

$$\frac{C\beta^N}{2^N} \geq .5$$

$$N \geq 5.93879704212366 \cdot 10^9.$$

In other words, we have to draw a string of length $\sim 6 \cdot 10^9$ to be 50% sure there is a run of 50 consecutive zeros.