

# COS 488 - Homework 6 - Question 1

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Let  $B_{32}(z)$  be the OGF for the class of binary strings that do not contain 32 consecutive 0's, so that

$$B_{32}(z) = \frac{1 - z^{32}}{1 - 2z + z^{33}}$$

by the formula given in the lecture. By the radius-of-convergence transfer theorem, we have that

$$[z^n]B_{32}(z) = \left(1 - \frac{1}{\beta_{32}^{32}}\right)\beta_{32}^n,$$

where  $\frac{1}{\beta_{32}}$  is the smallest root of  $1 - 2z + z^{33}$ , or approximately

$$0.500000000058.$$

Thus,

$$[z^n]B_{32}(z) \approx 0.999999999767 \times 1.999999999767^n.$$

The least value of  $n$  for which this quantity is less than  $\frac{2^n}{2}$ , which is the least value a string of random bits must be in order to be 50% sure that there are at least 32 consecutive 0's, is therefore

$$\approx 5.9541 \times 10^9.$$