COS 488 - Homework 6 - Question 2

Matt Tyler

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Let s represent the string "THE QUICK BROWN FOX JUMPED OVER THE LAZY DOG" which is a string consisting of 44 characters such that no suffix of the string is also a prefix of the string.
Let S_s be the combinatorial class representing the 32-key strings that do not contain s, and let T_s be the combinatorial class representing the 32-key strings that end in s and have no other occurrence of s. Then, S_s and T_s are disjoint, and adding a character to S_s gives a string in either S_s or T_s, so we have the construction

$$S_s + T_s = E + S_s \times \sum_{k=0}^{32} Z_k.$$

Furthermore, concatenating an element of S_s with s always gives an element of T_s, and each element of T_s is obtained uniquely in this way, so we also have the construction

$$S_s \times \{s\} = T_s.$$

This gives the OGF equations

$$S_s(z) + T_s(z) = 1 + 32zS_s(z) \text{ and } z^{44}S_s(z) = T_s(z),$$

which can be solved to give

$$S_s(z) = \frac{1}{1 - 32z + z^{44}}.$$

Then, by the same reasoning as was given in the lecture, the expected number of characters typed before the monkey hits upon s is

$$S_s\left(\frac{1}{32}\right) = \frac{1}{1 - 1 + \frac{1}{32^{44}}} = 32^{44} \approx 1.6850 \times 10^{66}.$$