COS 488 Problem Set #6 Question #1

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We know that the number of bitstrings without a run of p zeros is asymptotic to $c\beta^n$ where β is the root of largest magnitude of $x^{p+1} - 2x^p + 1 = 0$ and $c = \frac{\beta(1-\beta)(1-\beta^p)}{1-2\beta+\beta^{p+1}}$. When $p = 32$, if we let $\beta = 2 + \epsilon$ then we have $0 = (2+\epsilon)^{33} - 2(2+\epsilon)^{32} + 1 \approx 2^{32}\epsilon + 1$ so $\beta \approx 2 - 2^{-32}$. Moreover, $c \approx 1$. This gives that the proportion of bitstrings without 32 zeros is approximately $(\beta/2)^n = (1 - 2^{-33})^n$. We want this to be $1/2$. If $n = 2^{33}k$, then $(1 - 2^{-33})^{2^{33}k} \approx e^{-k}$, so $k = \log 2$ gives a good approximation. Hence, when bitstrings are of length at least $2^{33} \log 2 \approx 5.954 \times 10^9$ contain a run of 32 zeros with probability about $1/2$.