COS 488 Week 7: Q1

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We want the class of binary strings with no 11. A binary string with no 11 is either empty, 0, 1, or 10 followed by a binary string with no 11. Thus we have the symbolic construction:

$$B_{11} = E + Z_0 + (Z_1 + Z_1 \times Z_0)_{11}$$

giving

$$B_{11}(z) = 1 + z + (z + z^2)B_{11}(z)$$

giving the explicit generating function

$$B_{11}(z) = \frac{1+z}{1-z-z^2}$$

From AA08-Strings.pdf slide 7, we see that this gives the coefficient of z^N as $\frac{\phi^2}{\sqrt{5}}\phi^N$, where $\phi = \frac{1+\sqrt{5}}{2}$.
The coefficient of z^j is C_j, thus we want the sum

$$\sum_{j=0}^{L} \frac{\phi^2}{\sqrt{5}}\phi^j = \frac{\phi^2}{\sqrt{5}} \sum_{k=0}^{L} \phi^j = \frac{\phi^2}{\sqrt{5}} \frac{\phi^L - 1}{\phi - 1}$$

to be greater than or equal to 2^n, since we need words made from just the characters 0 and 1 to inject into our modified strings.
Note that $\lg 2n = \lg n + 1$ so $\lg(n-1) + 1 = \lg n - 1 + O(1) \implies \lg n - 1 = \lg n + O(1)$.

$$\frac{\phi^2}{\sqrt{5}} \frac{\phi^L - 1}{\phi - 1} \geq 2^n$$

and so taking the lg of both sides

$$\lg \frac{\phi^2}{\sqrt{5}} \frac{\phi^L - 1}{\phi - 1} = \lg(\phi^L - 1) + O(1) = \lg \phi^L + O(1)$$

(note that multiplying by constants inside the log leads to adding by constants outside)

$$L \lg \phi + O(1) \geq n$$

$$L(n) \geq n / \lg \phi + O(1)$$

and thus we have the desired identity:

$$L(n) \geq \lambda n + O(1).$$

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