

Miranda Moore  
COS 488/MAT 474  
Problem Set 7, Q&A

**Question** Write an EGF equation for the number of mappings with no fixed point, i.e., mappings where no atom is mapped to itself. Your answer may be left in terms of the function  $C(z) = ze^{C(z)}$ , the EGF for Cayley trees.

*Solution.* A mapping with no fixed point is a set of cycles of Cayley trees, but the cycles must have length 2 or greater. In symbols,

$$\mathcal{N} = SET(CYC_{\geq 2}(\mathcal{C})).$$

Translating this into an EGF, we get

$$\begin{aligned} N(z) &= \exp\left(\sum_{k \geq 2} \frac{C(z)^k}{k}\right) \\ &= \exp\left(\ln \frac{1}{1 - C(z)} - C(z)\right) \\ &= \frac{e^{-C(z)}}{1 - C(z)}. \end{aligned}$$