COS 488 - Homework 7 - Question 1

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Since $\mathcal{C}$ is the combinatorial class of all strings that do not contain the sequence 11, we have the following OGF equation from lecture (slide 21):

$$\mathcal{C}(z) = (1 + z)(1 + z\mathcal{C}(z)).$$

By solving this equation for $\mathcal{C}(z)$, we have

$$\mathcal{C}(z) = \frac{1 + z}{1 - z - z^2}.$$

By using partial fractions, we see that the number of allowed code-words of length n is

-0.5

$$[z^n]\mathcal{C}(z) = \alpha\varphi^n + \beta\phi^n$$

Should be F_{n+2}

where $\varphi = \frac{1+\sqrt{5}}{2}$ and $\phi = \frac{1-\sqrt{5}}{2}$ are the multiplicative inverses of the roots of $1 - z - z^2$. Therefore, since $|\phi| < 1$, a code of length L is allowable iff

$$2^n \leq \sum_{j=0}^{L} [z^n]\mathcal{C}(z) = \sum_{j=0}^{L} \alpha\varphi^j + O(1) = \beta\varphi^L + O(1)$$

for some constant β. By taking logarithms of both sides, this is true iff

$$n \leq L \log_2(\varphi) + O(1),$$

or

$$L \geq \frac{n}{\log_2(\varphi)} + O(1),$$

as desired.