COS 488 Problem Set #7 Question #1

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As shown in the book, the OGF for bitstrings not containing 11 is $\frac{1+z}{(1-2z)(1+z)+z^2} = \frac{1+z}{1-z-z^2}$ since the autocorrelation polynomial is $1+z$. By Theorem 4.1, since $1-z-z^2$ is satisfied by $-\varphi$ and φ^{-1} where $\varphi = \frac{1+\sqrt{5}}{2}$, we have that $[z^n]\frac{1+z}{1-z-z^2} \sim \frac{-\varphi(1+\varphi^{-1})}{-1-2\varphi^{-1}}\varphi^n = \frac{1}{\sqrt{5}}\varphi^{n+2}$. We wish to find a minimal λ so that $L(n) \equiv \lambda n + O(1)$ asymptotically. As there are 2^n strings of length n and they must be mapped injectively for the code to work, we in particular must have $\frac{1}{\sqrt{5}}\varphi^{\lambda n+2} > 2^n$.

$$\frac{1}{\sqrt{5}}\varphi^{\lambda n+2} = 2^n$$

$$-\frac{1}{2}\log_2 5 + (\lambda n + 2)\log_2 \varphi = n$$

$$\lambda n \log_2 \varphi = n - 2\log_2 \varphi + \frac{1}{2}\log_2 5$$

$$\lambda = \frac{n - 2\log_2 \varphi + \frac{1}{2}\log_2 5}{n \log_2 \varphi} \sim \frac{1}{\log_2 \varphi}$$

As a result, the most compress the code can be is asymptotically $L(n) \equiv \lambda n + O(1) \approx 1.44n + O(1)$