Test Question 2

David Luo

Question: How many ones, on average, are in an N-length bitstring with no two consecutive zeroes?

Solution

We construct the class of all binary bitstrings without 00, attaching a parameter to the ones, as

$$B = E + Z_0 + (uZ_1 + Z_0uZ_1)B$$

This results in the following OGF:

$$B(z,u) = 1 + z + (uz + uz^2)B(z,u)$$

$$B(z,u) = \frac{1+z}{1-uz-uz^2}$$

Now calculate the total count and cumulated cost:

$$[z^N]B(z,1) = [z^N]\frac{1+z}{1-z-z^2} = F_{N+2} = \frac{\varphi^2}{\sqrt{5}}\varphi^N$$

$$[z^N]B_u(z,1) = [z^N]\frac{z(z+1)^2}{(z^2+z-1)^2} = N\frac{\varphi^2}{\sqrt{5}}\varphi^{N-1}$$

Finally, divide to find the average:

$$\mu_N = \frac{[z^N]B_u(z,1)}{[z^N]B(z,1)} = N/\varphi$$