5/5

COS 488 Problem Set #8 Question #2

Tim Ratigan

April 13, 2017

First we will count the total number of such numbers. There are 45 choices for the location of the 1, then $\binom{44}{2}$ options for placing the 2 after then 1, then $\binom{42}{3}$ for the 3's, and so forth up to $\binom{9}{9}$. Multiplying these together, there are $\frac{45!}{1!2!3!4!5!6!7!8!9!}$ such numbers. Then, in each number, the probability that a given digit is $1 \leq i \leq 9$ is $\frac{i}{45}$, so the expected value of a digit is $\frac{1}{45}\sum_{i=1}^{9} i^2$. Finally, we multiply by the value of each of the digits $\sum_{i=0}^{44} 10^i$. This gives

$$\frac{45!}{1!2!3!4!5!6!7!8!9!}\frac{9\cdot 10\cdot 19}{45}\frac{10^{45}-1}{9}$$
$$= 45\ 875\ 559\ 600\ 006\ 153\ 219\ 084\ 769\ 286\ 399\ 999\ 999\ 999\ 999\ 954\ 124\ 440\ 399\ 993\ 846\ 780\ 915\ 230\ 713\ 600\ 000$$

This has a run of 9s because $\frac{44!}{1!2!3!4!5!6!7!8!9!}\cdot 10\cdot 19 = 45\ 875\ 559\ 600\ 006\ 153\ 219\ 084\ 769\ 286\ 400\ 000$ has 35 digits, so when we multiply this by $10^{45}-1$, we subtract a number with at least 45 zeroes by a number with only 35 digits, resulting in a sequence of 9s in the middle.