COS 488 Problem Set #8 Exam Question

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April 11, 2017

Question: Write an expression for the OBGF for the number of vertices of degree d in a tree of size n. Solve for $d = 1$.

Answer: We construct a combinatorial class for trees counting vertices of degree d, $\mathcal{G}_d$. Then we have the cost function $\sum_g z^{|g|}u^{\deg_k g}$ where $\deg_k$ is the number of degree-k vertices. Then we have the construction

$$\begin{aligned}
\mathcal{G}_d &= \mathcal{Z} \times SEQ(\mathcal{G}_d) + (u-1) \times \mathcal{Z} \times SEQ_d(\mathcal{G}_d)\\
G_d(z,u) &= \frac{z}{1-G_d(z,u)} + (u-1)zG_d(z,u)^d\\
G_d(z,u)(1-G_d(z,u)) &= z + (u-1)zG_d(z,u)^d(1-G_d(z,u))\\
0 &= (1-u)zG_d(z,u)^{d+1} - (1-u)zG_d(z,u)^d + G_d(z,u)^2 - G_d(z,u) + z
\end{aligned}$$

At $d = 1$, we have

$$\begin{aligned}
0 &= ((1-u)z+1)G_1(z,u)(G_1(z,u)-1) + z\\
G_1(z,u) &= \frac{(1-u)z+1-\sqrt{((1-u)z+1)^2-4z((1-u)z+1)}}{2((1-u)z+1)}\\
&= \frac{1}{2}\left(1-\sqrt{1-\frac{4z}{(1-u)z+1}}\right)
\end{aligned}$$