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COS 488

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A "supernecklace" of the 3rd type is a labeled cycle of cycles (see page 125). Draw all the supernecklaces of the 3rd type of size N for $N = 1, 2, 3$, and 4. Then develop an asymptotic estimate of the number of supernecklaces of size N by showing that

$$[z^n]ln(\frac{1}{1-ln\frac{1}{1-z}}) \sim \frac{1}{n}(1-e^{-1})^{-n}$$

-1 Drawings?

Knowing that we can find terms using derivatives with the formula $[z^n]zS'(z) = ns_n$, we can substitute in the above function to get our terms.

$$[z^n]z\frac{d}{dz}ln(\frac{1}{1-ln\frac{1}{1-z}}) = ns_n$$

$$[z^n]\frac{z}{(1-z)(1-ln\frac{1}{1-z})} = ns_n$$

$$s_n = \frac{1}{n}[z^n]\frac{z}{(1-z)(1-ln\frac{1}{1-z})}$$

Now invoke the meromorphic transfer theorem:

$$\alpha = 1 - \frac{1}{e}$$

$$\beta = \frac{1}{1-\frac{1}{e}}$$

$$h_{-1} = \frac{-f(\alpha)}{g'(\alpha)} = 1 - \frac{1}{e}$$

$$c = \frac{h_{-1}}{\alpha} = 1$$

$$\frac{f(z)}{g(z)} = c\beta^n = (1 - \frac{1}{e})^{-n}$$

Plug this back in to get the result:

$$s_n = \frac{1}{n}(1-e^{-1})^{-n}$$