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COS 488

Integrate $f(z) = z$ on a triangle with vertices at $A(1,1)$, $B(1,0)$, $C(0,0)$ in the complex plane.

Solution:

Since z is analytic in Ω and triangles are closed loops, then $\int_{\lambda} z dz = 0$.

We can go leg-by-leg and show that everything adds up to zero. Let's call the leg from A to B leg 1, B to C leg 2, and C to A leg 3.

$$\int_{L_1} z dz = \int_1^0 (1 + iy)idy = iy - \frac{y^2}{2} \Big|_{y=1,0} = \frac{1}{2} - i$$

$$\int_{L_2} z dz = \int_1^0 x dx = \frac{x^2}{2} \Big|_{x=1,0} = -\frac{1}{2}$$

$$\int_{L_3} z dz = \int_0^1 x dx + i + \int_0^1 (0 + iy)idy = \frac{x^2}{2} \Big|_{x=0,1} + i - \frac{y^2}{2} \Big|_{y=0,1} = i$$

$$\int_{\lambda} z dz = \int_{L_1+L_2+L_3} z dz = \frac{1}{2} - i - \frac{1}{2} + i = 0$$