COS 488: AC week 3 Q1

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The pictures are drawn at the bottom.
We begin by taking the derivative of the function, which gives:

$$\frac{1}{1-z}\frac{1}{1-\ln\frac{1}{1-z}}$$

we now apply the transfer theorem from slide 59 of the poles lecture:
we let $f(x) = \frac{1}{1-z}$ and $g(x) = 1 - \ln\frac{1}{1-z}$. We note that are poles at 1 and when

$$1 - \ln\frac{1}{1-z} = 0$$

$$e^1 = \frac{1}{1-z}$$

$$z = 1 - 1/e$$

we note that the latter root is closer to the origin, so we have $\alpha = 1 - 1/e$ with order M = 1. Applying the theorem we get:

$$\frac{(-1)(1)\frac{1}{1-(1-e^{-1})}}{(1-e^{-1})^1(\frac{-1}{1-(1-e^{-1})})}(\frac{1}{1-e^{-1}})^n n^0 = (1-e^{-1})^{-(n+1)}$$

adjusting for the earlier derivative, we take the $n-1^{st}$ coefficient, and divide by n, giving the desired:

$$[z^n]h(z) \sim \frac{1}{n}(1-e^{-1})^{-n}$$

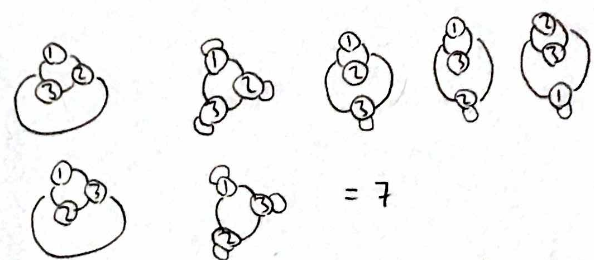
Now to develop the asymptotic estimate of the number of them of size n, we multiply by n! using Stirling's approximation, giving:

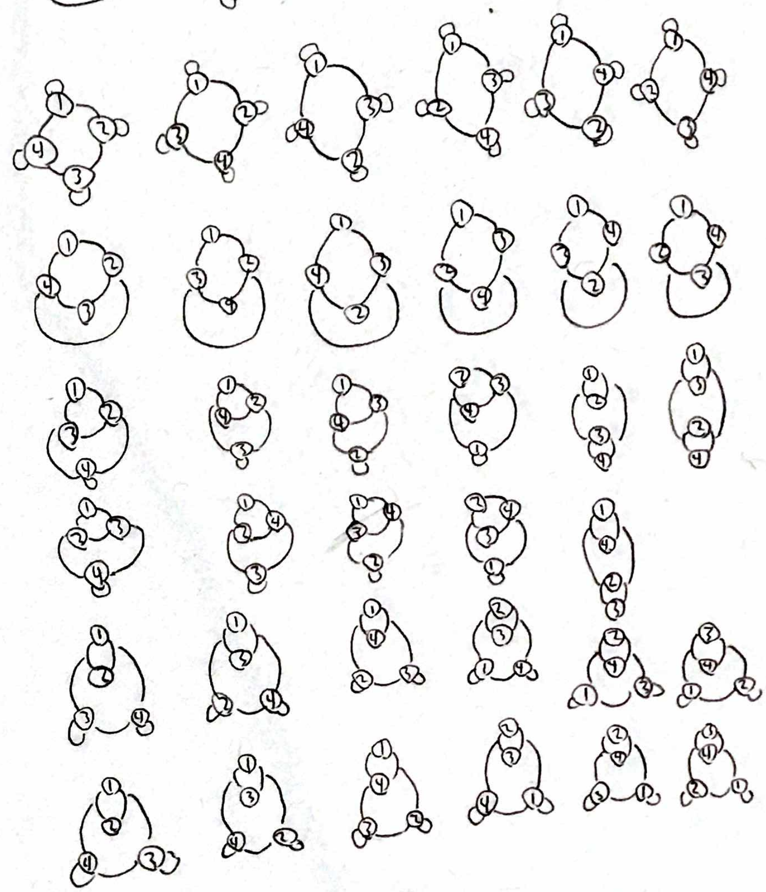
$$\sqrt{\frac{2\pi}{n}}(\frac{n}{e-1})^n$$

Plugging in n = 4, we get 36.8, which is very close to the 35 we get below.

$n=1$:  $=1$

$n=2$:  $=2$

$n=3$:  $=7$

$n=4$: 

$=35$