COS 488 - Homework 9 - Program IV.1

Matt Tyler

5/5

```java
/*************************************************************************
 *  Author:    Matt Tyler
 *
 *  Description: Computes an estimations of the percentage of permutations
 *  having so singleton or doubleton cycles. Takes two command-line arguments
 *  representing the number of items in the permutations and the number of
 *  random permutations to check.
 *
 *  Usage example:
 *      java-algs4 NoShortCycles 10 1000000
 *      java-algs4 NoShortCycles 20 1000000
 *************************************************************************/

import edu.princeton.cs.algs4.StdRandom;
import edu.princeton.cs.algs4.StdOut;

public class NoShortCycles
{
    private static boolean ShortCycles(int[] perm)
    {
        for (int i = 0; i < perm.length; i++)
        {
            if (perm[perm[i]] == i) return true;
        }
        return false;
    }

    public static void main(String[] args)
    {
        int N     = Integer.parseInt(args[0]);
        int total = Integer.parseInt(args[1]);

        int count = 0;
        for (int i = 0; i < total; i++)
        {
            int[] perm = new int[N];
            for (int j = 0; j < N; j++) perm[j] = j;
            StdRandom.shuffle(perm);
            if (ShortCycles(perm)) count++;
        }

        StdOut.printf("%f", 1 - ((double) count) / total);
    }
}
```

With 10,000,000 iterations, this outputs approximately 22.32% both when $N = 10$ and when $N = 20$.
A permutation with neither singleton nor doubleton cycles can be constructed as a set of cycles of lengths at least 3, so the EGF for the permutations with neither singleton nor doubleton cycles is

$$\Omega(z) = e^{\ln(1/(1-z)) - z^2/2 - z} = \frac{e^{-z^2/2-z}}{1-z}.$$

Since 1 is the unique pole of $\Omega(z)$ and it has order 1, we have by the transfer theorem in lecture that the percentage of permutations of size N with neither singleton nor doubleton cycles is

$$\begin{aligned}\frac{N![z^N]\Omega(z)}{N!} &= [z^N]\Omega(z) \\ &\sim -\frac{e^{-1^2/2-1}}{-1}1^N \\ &= e^{-3/2}.\end{aligned}$$

Since $e^{-3/2} \sim 22.31\%$, this agrees with the values computed emperically.