COS 488 Problem Set #9 Question #1

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The construction is straightforward; the combinatorial class of cycles of cycles is simply

-1 Drawings?

$$\begin{aligned}
\mathcal{T} &= CYC(CYC(Z)) \\
T(z) &= \log\left(\frac{1}{1-\log\left(\frac{1}{1-z}\right)}\right) \\
&= -\log(1+\log(1-z)) \\
T'(z) &= \frac{1}{1+\log(1-z)}\frac{1}{1-z}
\end{aligned}$$

This function has two poles within $|\text{Im}(z)| \leq \pi$, one at $1-z=0 \implies z=1$, and one at $1+\log(1-z)=0 \implies 1-z=e^{-1} \implies z=1-e^{-1}$. Note that

$$\begin{aligned}
\lim_{z\to 1-e^{-1}}(z-(1-e^{-1}))T'(z) &= \frac{\frac{\mathrm{d}z-(1-e^{-1})}{\mathrm{d}z}}{\frac{\mathrm{d}(1+\log(1-z))}{\mathrm{d}z}}\Big|_{z=1-e^{-1}} \\
&= \frac{1}{-\frac{1}{1-z}}\Big|_{z=1-e^{-1}} \\
&= z-1|_{z=1-e^{-1}} \\
&= -e^{-1}
\end{aligned}$$

As a result, $1-e^{-1}$ is a pole of order 1, and is the pole of smallest magnitude. By the transfer theorem for meromorphic functions, we have

$$[z^n]T'(z) \sim c(1-e^{-1})^{-n}$$

where

$$\begin{aligned}
c &= -\frac{1}{(1-e^{-1})\frac{\mathrm{d}(1+\log(1-z))(1-z)}{\mathrm{d}z}\Big|_{z=1-e^{-1}}} \\
&= -\frac{1}{1-e^{-1}}\frac{1}{1+\log(1-z)-1}\Big|_{z=1-e^{-1}} \\
&= \frac{1}{1-e^{-1}} \\
\implies [z^n]T'(z) &\sim (1-e^{-1})^{-n-1} \\
[z^{n+1}]T(z) &\sim \frac{1}{n+1}(1-e^{-1})^{-n-1} \\
[z^n]T(z) &\sim \frac{1}{n}(1-e^{-1})^{-n}
\end{aligned}$$