

# COS 488 Problem Set #9 Test Example Question

Tim Ratigan

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## Question

Give an asymptotic estimate for the number of cycles of nonempty collections of labelled objects.

## Answer

The construction for this is given by  $\mathcal{T} = \text{CYC}(\text{SET}_{\geq 1}(\mathcal{Z}))$  which translates to  $T(z) = \log \frac{1}{1-(e^z-1)} = -\log(2-e^z)$ . If we differentiate, we obtain the meromorphic function  $\frac{e^z}{2-e^z}$ . This has a simple pole at  $z = \log 2$ , so by the transfer theorem the coefficient of  $z^n$  is asymptotic to  $\frac{1}{(\log 2)^n} \cdot \frac{-e^{\log 2}}{-\log 2 \cdot e^{\log 2}} = \frac{1}{(\log 2)^{n+1}}$ . Integrating implies that the coefficient of  $z^{n+1}$  in the original series is  $\frac{1}{(n+1)(\log 2)^{n+1}}$ , and therefore  $[z^n]T(z) \sim \frac{1}{n(\log 2)^n}$ .